

الحساب العددي
الجزور المربعة



التمرين الأول
أبسط

$$\sqrt{16} = 4 ; \sqrt{81} = 9 ; (-\sqrt{3})^2 = 3 ; \sqrt{25} = 5 ; \sqrt{50} = 5\sqrt{2}$$

$$\frac{1}{(-\sqrt{7})^{-4}} = (-\sqrt{7})^4 = 49 ; \sqrt{0.001} = \sqrt{10^{-3}} = \frac{1}{\sqrt{10^3}} = \frac{1}{10\sqrt{10}} = \frac{\sqrt{10}}{100}$$

$$\sqrt{\frac{16}{9}} = \frac{4}{3} ; \frac{\sqrt{25}}{\sqrt{81}} = \frac{5}{9} ; (\sqrt{7})^8 = 7^4 = 2401$$

التمرين الثاني
أحسب

$$a = \sqrt{25} + \sqrt{16} = 5 + 4 = 9 ; b = \sqrt{(-5)^2} - 4(\sqrt{3})^2 = 5 - 4 \times 3 = 5 - 12 = -7 ;$$

$$c = \sqrt{100} - \sqrt{1} - (4\sqrt{3})^2 = 10 - 1 - 48 = -39 ; d = \sqrt{\frac{9}{121}} \times \left(\frac{\sqrt{2}}{\sqrt{3}}\right)^2 = \frac{\cancel{3}}{11} \times \frac{2}{\cancel{3}} = \frac{2}{11}$$

$$e = \left(\sqrt{\frac{2}{3}}\right)^2 \times \left(\frac{1}{\sqrt{3}}\right)^{-2} = \frac{2}{\cancel{3}} \times \cancel{3} = 2 ;$$

$$f = \sqrt{7} \times \sqrt{3^2 + 5^2 + 9 - 15} = \sqrt{7} \times \sqrt{9 + 25 + 9 - 15} = \sqrt{7} \times \sqrt{28} = \sqrt{7} \times 2\sqrt{7} = 14$$

$$g = \sqrt{0.0004} + \sqrt{10^{-6}} = \sqrt{4 \times 10^{-4}} + \sqrt{(10^{-3})^2} = 2 \times 10^{-2} + 10^{-3} = \frac{2}{100} + \frac{1}{1000} = 0.021$$

$$h = \sqrt{\frac{0.49}{0.01}} + \frac{\sqrt{11}}{2} \sqrt{44} = \sqrt{49} + \frac{\sqrt{11}}{\cancel{2}} \times \cancel{2} \sqrt{11} = 7 + 11 = 18$$

$$i = \frac{\sqrt{2} \times \sqrt{3} \times \sqrt{6}}{(-\sqrt{20}) \times \sqrt{45}} = \frac{\sqrt{2} \times \sqrt{3} \times \sqrt{6}}{-2\sqrt{5} \times 3\sqrt{5}} = \frac{\cancel{6}}{-\cancel{6} \times 5} = -\frac{1}{5}$$

$$j = \sqrt{6 + 3\sqrt{100}} = \sqrt{6 + 3 \times 10} = \sqrt{6 + 30} = \sqrt{36} = 6$$

$$k = \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{4}}}} = \sqrt{2 + \sqrt{2 + \sqrt{2 + 2}}} = \sqrt{2 + \sqrt{2 + \sqrt{4}}}$$

$$= \sqrt{2 + \sqrt{2 + 2}} = \sqrt{2 + \sqrt{4}} = \sqrt{2 + 2} = \sqrt{4} = 2$$

التمرين الثالث

$$B_1 = 2\sqrt{12} + 3\sqrt{27} - 2\sqrt{48} \quad ; \quad B_2 = \sqrt{0.50} - \sqrt{0.98} + 3\sqrt{0.18} - 4\sqrt{0.72}$$

$$= 2 \times 2\sqrt{3} + 3 \times 3\sqrt{3} - 2 \times 4\sqrt{3} \quad ; \quad B_2 = 0.5\sqrt{2} - 0.7\sqrt{2} + 3 \times 0.3\sqrt{2} - 4 \times 0.6\sqrt{2}$$

$$= 4\sqrt{3} + 9\sqrt{3} - 8\sqrt{3} \quad ; \quad B_2 = 0.5\sqrt{2} - 0.7\sqrt{2} + 0.9\sqrt{2} - 2.4\sqrt{2}$$

$$= (4+9-8)\sqrt{3} \quad ; \quad B_2 = (0.5-0.7+0.9-2.4)\sqrt{2}$$

$$\boxed{B_1 = 5\sqrt{3}} \quad ; \quad \boxed{B_2 = -1.7\sqrt{2}}$$

$$B_3 = \frac{2\sqrt{45} - 3\sqrt{80} + 4\sqrt{5}}{20\sqrt{5}} \quad ; \quad B_4 = 2 - (9 + \sqrt{25}) - 4(\sqrt{7} - 1) + \sqrt{6^2 \times 7}$$

$$= \frac{2 \times 3\sqrt{5} - 3 \times 4\sqrt{5} + 4\sqrt{5}}{20\sqrt{5}} \quad ; \quad B_4 = 2 - 9 - 5 - 4\sqrt{7} + 4 + 6\sqrt{7}$$

$$= \frac{6\sqrt{5} - 12\sqrt{5} + 4\sqrt{5}}{20\sqrt{5}} \quad ; \quad \boxed{B_4 = -8 + 2\sqrt{7}}$$

$$= \frac{(6-12+4)\sqrt{5}}{20\sqrt{5}}$$

$$B_3 = \frac{-2}{20} = -\frac{1}{10}$$

$$B_5 = \frac{2}{\sqrt{20}} - \frac{3}{\sqrt{45}} + \frac{2\sqrt{2}}{\sqrt{80}} - \frac{1}{2}\sqrt{\frac{2}{5}} \quad ; \quad B_6 = \sqrt{\frac{7}{3}} + 4\sqrt{\frac{63}{75}} - 2\sqrt{\frac{28}{27}}$$

$$B_5 = \frac{\cancel{2}}{\cancel{2}\sqrt{5}} - \frac{\cancel{3}}{\cancel{3}\sqrt{5}} + \frac{\cancel{2}\sqrt{2}}{\cancel{4}\sqrt{5}} - \frac{1}{2}\sqrt{\frac{2}{5}} \quad ; \quad B_6 = \sqrt{\frac{7}{3}} + 4 \times \frac{3}{5}\sqrt{\frac{7}{3}} - 2 \times \frac{2}{3}\sqrt{\frac{7}{3}}$$

$$B_5 = \frac{1/\cancel{2}}{\sqrt{5}} - \frac{1/\cancel{3}}{\sqrt{5}} + \frac{1/\cancel{2}\sqrt{2}}{\cancel{2}\sqrt{5}} - \frac{1/\cancel{2}\sqrt{2}}{\cancel{2}\sqrt{5}} \quad ; \quad B_6 = \left(1 + \frac{12}{5} - \frac{4}{3}\right)\sqrt{\frac{7}{3}}$$

$$\boxed{B_5 = 0} \quad ; \quad \boxed{B_6 = \frac{31}{15}\sqrt{\frac{7}{3}}}$$

التمرين الرابع

أحسب

$$F_2 = (-\sqrt{7} + 4\sqrt{5})(-\sqrt{7} - 4\sqrt{5})$$

$$F_2 = (-\sqrt{7})^2 - (4\sqrt{5})^2$$

$$\boxed{F_2 = 7 - 80 = -73}$$

$$F_1 = (2\sqrt{3} - 5)^2 - 2(2\sqrt{3} + 5)^2$$

$$F_1 = 12 - 20\sqrt{3} + 25 - 24 - 40\sqrt{3} - 50$$

$$\boxed{F_1 = -37 - 60\sqrt{3}}$$

$$F_4 = (\sqrt{28} + \sqrt{7} - \sqrt{32})(\sqrt{63} + 2\sqrt{8})$$

$$F_4 = (2\sqrt{7} + \sqrt{7} - 4\sqrt{2})(3\sqrt{7} + 4\sqrt{2})$$

$$F_4 = (3\sqrt{7} - 4\sqrt{2})(3\sqrt{7} + 4\sqrt{2})$$

$$F_4 = (3\sqrt{7})^2 - (4\sqrt{2})^2$$

$$F_4 = 63 - 32$$

$$\boxed{F_4 = 31}$$

$$F_3 = (\sqrt{1000} + 10\sqrt{6} - \sqrt{360} - 6\sqrt{6})(\sqrt{10} - \sqrt{6})$$

$$F_3 = (10\sqrt{10} + 10\sqrt{6} - 6\sqrt{10} - 6\sqrt{6})(\sqrt{10} - \sqrt{6})$$

$$F_3 = (4\sqrt{10} + 4\sqrt{6})(\sqrt{10} - \sqrt{6})$$

$$F_3 = 4(\sqrt{10} + \sqrt{6})(\sqrt{10} - \sqrt{6})$$

$$F_3 = 4(10 - 6)$$

$$\boxed{F_3 = 4 \times 4 = 16}$$

$$F_6 = \left(\frac{\sqrt{5}}{\sqrt{6} + \sqrt{5}} \right)^{-2}$$

$$F_6 = \left(\frac{\sqrt{6} + \sqrt{5}}{\sqrt{5}} \right)^2$$

$$\boxed{F_6 = \frac{11 + 2\sqrt{30}}{5}}$$

$$F_7 = \sqrt{\frac{9a^4}{a^6b}} \times \sqrt{\frac{(ab^3)^2 a^2}{(b^2)^2}}$$

$$F_7 = \frac{3a^2}{a^3\sqrt{b}} \times \frac{a^2b^3}{b^2}$$

$$\boxed{F_7 = \frac{3ab}{\sqrt{b}} = 3a\sqrt{b}}$$

$$F_5 = \frac{2}{\sqrt{5}-1} + \frac{3}{3+\sqrt{5}} - \frac{11-\sqrt{5}}{4}$$

$$F_5 = \frac{2(\sqrt{5}+1)}{5-1} + \frac{3(3-\sqrt{5})}{9-5} - \frac{11-\sqrt{5}}{4}$$

$$F_5 = \frac{2\sqrt{5}+2}{4} + \frac{9-3\sqrt{5}}{4} - \frac{11-\sqrt{5}}{4}$$

$$F_5 = \frac{2\sqrt{5}+2+9-3\sqrt{5}-11+\sqrt{5}}{4}$$

$$\boxed{F_5 = 0}$$

التمرين الخامس

لدينا

$$\frac{30}{6-\sqrt{6}} - \frac{6}{3\sqrt{2}-2\sqrt{3}} + \frac{\sqrt{6}}{\sqrt{3}-\sqrt{2}} = \frac{30(6+\sqrt{6})}{36-6} - \frac{6(3\sqrt{2}+2\sqrt{3})}{18-12} + \frac{\sqrt{6}(\sqrt{3}+\sqrt{2})}{3-2}$$

$$= \frac{\cancel{30}(6+\sqrt{6})}{\cancel{30}} - \frac{\cancel{6}(3\sqrt{2}+2\sqrt{3})}{\cancel{6}} + \frac{\sqrt{6}(\sqrt{3}+\sqrt{2})}{1}$$

$$= 6 + \sqrt{6} - 3\sqrt{2} - 2\sqrt{3} + \sqrt{18} + \sqrt{12}$$

$$= 6 + \sqrt{6} - \cancel{3\sqrt{2}} - \cancel{2\sqrt{3}} + \cancel{3\sqrt{2}} + \cancel{2\sqrt{3}}$$

$$\frac{30}{6-\sqrt{6}} - \frac{6}{3\sqrt{2}-2\sqrt{3}} + \frac{\sqrt{6}}{\sqrt{3}-\sqrt{2}} = 6 + \sqrt{6}$$

$$\frac{30}{6-\sqrt{6}} - \frac{6}{3\sqrt{2}-2\sqrt{3}} + \frac{\sqrt{6}}{\sqrt{3}-\sqrt{2}} = 6 + \sqrt{6}$$

$$\begin{aligned} \frac{10\sqrt{3}}{2\sqrt{3}-\sqrt{2}} &= \frac{10\sqrt{3}(2\sqrt{3}+\sqrt{2})}{12-2} \\ &= \frac{\cancel{10}\sqrt{3}(2\sqrt{3}+\sqrt{2})}{\cancel{10}} \end{aligned}$$

$$\frac{10\sqrt{3}}{2\sqrt{3}-\sqrt{2}} = 6 + \sqrt{6}$$

$$\frac{30}{6-\sqrt{6}} - \frac{6}{3\sqrt{2}-2\sqrt{3}} + \frac{\sqrt{6}}{\sqrt{3}-\sqrt{2}} = \frac{10\sqrt{3}}{2\sqrt{3}-\sqrt{2}}$$

إذن

التمرين السادس

$$\frac{\sqrt{ab(c^2+4)}}{a+b} = 1 \quad \text{لنبين أن}$$

$$\begin{aligned} \frac{\sqrt{ab(c^2+4)}}{a+b} &= \frac{\sqrt{ab\left(\frac{a}{b} + \frac{b}{a} - 2 + 4\right)}}{a+b} \\ &= \frac{\sqrt{a^2 + b^2 + 2ab}}{a+b} \\ &= \frac{\sqrt{(a+b)^2}}{a+b} \\ &= \frac{a+b}{a+b} \end{aligned}$$

$$\frac{\sqrt{ab(c^2+4)}}{a+b} = 1$$